SOME NOTES ON POWER, WINNING AND RATIONAL LEGITIMACY

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A common notion in the formal treatment of politics is that we can attribute a certain amount of 'power' to each member of a group which collectively can reach decisions of some kind. It's then normally assumed (without much explicit argument) that if you have more power (so defined) in a decision-making group you're better off than if you have less.

The best known of these power indexes is the Shapley-Shubik index of power in a voting body (or a collection of voting bodies). The 'power' of the members of a committee is assumed to be divisible between them and always to sum to unity. The general idea is to imagine everybody voting in favour of some measure, in every possible order, and then to ask in what proportion of all possible permutations is a given actor pivotal. The 'pivotal' actor is the one whose vote pushes the measure over from losing to winning.

An alternative measure, which also sums the individuals' 'powers' to unity and produces roughly comparable results is due to Banzhaf and asks in what proportion of all possible minimal winning combinations (not permutations--order doesn't matter here) a given voter's defection would be decisive in changing it from winning to losing.

In either case it's not at all clear what the significance of this 'power' is or why one should identify an individual actor's power on this index with the value of the voting game to him. Shapley and Shubik try to offer some meaning for it by asking us to imagine that the actors vote on the issue in question in decreasing order of enthusiasm for it. They then say that if the supporters of the motion were buying votes they would go first for the 'no' voter least opposed to the measure and offer him a bit, then the next most and offer him a bit more, until they got to the pivotal voter (i.e. the median enthusiast in a simple committee) who would get the most since he would be the most opposed to the measure of all those whose votes are needed to be bought. But first of all, the change from voting yes in random order to voting yes or no in order of decreasing enthusiasm for the measure seems to introduce a different game. And second, it's not at all obvious why the 'value of the game' is to be identified with the private payoff given to induce a voter inclined to vote against the measure to switch his vote so as to vote in favour of it. How come there are varying degrees of enthusiasm for the measure, anyway? Presumably because people expect different benefits (psychic or material) from the public policy which will be enacted if the measure passes. Unless those in favour have some reason for being in favour derived from the nature of the measure itself, it is inexplicable why they would be prepared to dig into their pockets so as to provide private benefits in the form of side-payments to induce reluctant voters to switch. The value of a favourable outcome on the issue is presumably large enough to leave those most in favour with a net balance after providing the side-payments.

This could be analysed further to allow for the possibility of counterbids from those who stand to lose if the measure is passed. And we can call the whole analysis into doubt by pointing out that the situation is one where the public policy is a collective good and that any given individual in a large body has only a small chance of changing the outcome, whereas the side-payment for voting a certain way is a definite private gain. So even if you expected to be seriously damaged by a measure, it would be worth taking a bribe to vote for it that was much less than the loss you'd sustain from its passage so long as you thought that the measure was very likely either to pass even if you voted against it or to fail even if you voted for it.

Another way of attacking the zero-sum measures of power is to show that they are capable of producing absurd results in some cases. Consider Brams's example on p. 181 of Game Theory and Politics where there are three players with weights of 3, 2 and 2, and where 5 votes out of 7 are needed for a majority decision. Call the players with 3 votes A and the other two B and C. Under normal circumstances the Shapley-Shubik values are 2/3 for A and 1/6 apiece for B and C and the Banzhaf values are 3/5 for A and 1/5 apiece for B and C; but if for some reason B and C never agree it becomes 1/2, 1/4, 1/4 on both indices.

On the Banzhaf criterion, this is a result of the fact that the coalitions AB and AC remain possible but the coalition A, B and C is ruled out as something that can never form. Thus, A can bring down two coalitions (i.e. make them less than winning) while B and C can bring down one each. Thus, there are four possible ways of bringing down a coalition, two involving A and one each B and C. Hence the scores A = 1/2, B = 1/4, C = 1/4. If A, B and C can form, however, we have an extra way in which a coalition can be brought down: by A's withdrawal from this 'grand coalition'. (Neither B nor C can bring it down by withdrawing from it since it would still have 5 votes.) Thus, on the Banzhaf index there would be 5 ways in which a coalition could be brought down, in 3 of which A figures, leaving one apiece for B and C.

On the Shapley-Shubik index, it is a matter of orderings in which each is pivotal (i.e. makes up the required majority of 5 votes). A$\underline{\text{B}}$C, A$\underline{\text{C}}$B, B$\underline{\text{A}}$C and C$\underline{\text{A}}$B are available permutations where B and C disagree, making A pivotal in 2 out of 4 and B and C pivotal in one each. But if B and C do not necessarily disagree, there are also available BC$\underline{\text{A}}$ and CB$\underline{\text{A}}$, which add two more permutations where A is pivotal. Thus, A would be pivotal in 4 cases out of 6 (2/3) and B and C one each out of six (1/6). Yet the conclusion, that the power of B and C has increased, and that 'there is an incentive for them to quarrel and increase their share of the voting power' is manifestly absurd. For the situation now is one where the 3 player always has a majority of 5 for what he favours, since there will be 2 votes in favour of each side automatically. Thus he is in effect a dictator. Brams says that although he has called this a paradox, it really shows us unexpected things about the nature of power. I think this is nonsense. What it shows us is that there's something wrong with the index.

My suggestion is that what you're interested in is the probability that when you're in favour of something the outcome will be positive and when you're against it the outcome will be negative. Thus, in the case analysed by Brams, A's probability of getting the outcome that he wants with B and C always quarrelling is unity: he can be sure that there will always be two votes to add to his own three, whichever way he goes, making the required majority of five out of seven votes. But if B and C may sometimes vote together, they can prevent A from getting an outcome he wants (though with a 4-3 majority they can't impose an outcome either -- all they can do is deadlock the issue). Therefore A is worse off with B and C voting independently than with B and C always voting on opposite sides. If A is <u>against</u> something, he can block it either way (his three votes are, as we've noted, enough to stop B and C from gaining an adequate majority against him) but if A is <u>for</u> something, he may not be able to get it if B and C vote independently whereas if they always vote on opposite sides he knows that he can always get it.

The common sense view is indeed that so far from B and C being better off if they always vote on <u>opposite</u> sides they would be better off if they always vote on the <u>same</u> side. This belief would be borne out by the present line of analysis. If B and C always vote together, the game reduces to one in which there is a bloc of 3 votes (A) and a bloc of 4 votes (B + C) and in which each has a veto on collective decisions since 5 votes are needed for a valid decision. A can get an outcome he wants only when B and C agree; and they are in an exactly equivalent position.

Suppose we want to quantify the ability of each player to get the outcome he wants under each of these three conditions (B and C always disagree, B and C always agree, B and C vote independently). To do so we have to stipulate some probability that if voters vote independently of one another they will vote the same way. Let us make it .5. Then we can analyse the three cases as follows:

(1) B and C always disagree; each agrees with A half the time.

A gets the outcome he wants all the time

B and C each get the outcome they want half the time.

(2) B and C always agree; they both agree with A half the time.

If A is Yes on an issue, B and C will agree half the time, and the outcome will be Yes.

If A is No on an issue, the outcome will be No whatever B and C want.

Let us suppose that issues on which A is Yes and No come up equally often

Then the long run expectation of A is 1/2(1/2 + 1) = 3/4.

The same analysis exactly covers B and C.

(3) B and C agree half the time with each other and with A.

If A Yes:			<u>Outcome Yes</u>
	B Yes 1/2		1/2
	B No	C Yes 1/2 x 1/2	1/4
			3/4

If A No: outcome always no.

Supposing issues on which A is Yes and No are equally likely to come up, A's expectation is 1/2(3/4 + 1) = 7/8.

If B Yes:			<u>Outcome Yes</u>
	A Yes 1/2		1/2
If B No:			<u>Outcome No</u>
	A No 1/2		1/2
	B Yes	C No 1/2 x 1/2	1/4
			3/4

Average expectation for B: 1/2(1/2 + 3/4) = 5/8.

C is exactly equivalent in position to B so it has the same expectation.

In summary form:

	Expectation of getting desired outcome		
	A	B	C
B and C always disagree	1	1/2	1/2
B and C are independent	7/8	5/8	5/8
B and C always agree	3/4	3/4	3/4

If we ask what use the whole enterprise of asking about the expected payoffs from membership in a decision-making body might be, the most obvious answer, I suppose, is that it might suggest whether or not it would be in your interests to join it or, if you're already a member, to try to leave it or make an effort to change the rules. We can say that this sort of power calculation will give us a theory of the way rational people would behave. It can be converted into a predictive theory inasfar as we are willing to predict that people will behave rationally.

Notice that this idea -- 'rational legitimacy' as Ronald Rogowski calls it in a book with that title -- is concerned with support for an institution purely as a function of power within it. It leaves out any kind of sentimental or principled attachment someone might have to a unit of some kind, or to a decision-making method of some kind, irrespective of its effects on his power. It's thus inherently limited, but

(1) it can be used as a benchmark -- people put forward fancy theories of legitimacy to support or attack institutions but it may be that these are self-serving. If we find a high correlation between power and satisfaction,that's interesting and suggestive.

(2) Sometimes people (or, even more, representatives of political entities) are quite explicit about their interest in power within larger decision-making groups. Examples: states at the US Constitutional Convention of 1789 or Daniel P. Moynihan's idea of 'the U.S. in opposition' in the United Nations General Assembly. I shall therefore take these as examples.

The difficulty raised by Rogowski's treatment of the subject in _Rational Legitimacy_ is fundamental. Since I shall dissent from him at a fundamental level, I shan't follow up the rest of his discussion, though it contains material of some interest.

The fundamental point is: _what_ is it about the membership of some group that makes for support on a basis of 'rational legitimacy'. So far I've loosely said 'power'. But is this exactly right? I don't think so.

Rogowski does take the line that 'power' is the thing. He offers as an index of power what he calls his 'coefficient of unique determination'. There's no need to go into the way that this is derived. It's quite cumbersome and appears to have no advantages over the Shapley-Shubik or Banzhaf indexes. (Oddly enough, he doesn't discuss its relation to them and suggest why it's better. In fact, he doesn't show any awareness of them in the text.) The crucial point about it is that it shares with them the feature that the object is to say something about the probability for any individual (or bloc) of being _decisive_, that is to say, changing the outcome from yes to no or no to yes. And these probabilities sum to unity for all the players taken together.

I want to suggest that in looking at some decision-making institution the most important question to ask (from the point of view of 'rational legitimacy') is not 'How likely am I to be decisive?' but 'How likely am I to be happy with the results?' (If it's a simple yes/no choice this reduces to: 'How likely is it that the side of the question I am in favour of will win?') More generally, we have to allow for multiple possible outcomes of decision-making and make the question: 'How far up my preference-schedule for outcomes do I expect the decisions reached to be?'

Let me take an example to illustrate the difference between the orthodox approach in terms of 'decisiveness' and the alternative that I'm proposing. Suppose that you're an inhabitant of Northern Ireland and a Constitutional Convention is to be held to decide on the future constitution of the province -- should it be a sovereign state, should it be autonomous within the UK, should it be integrated into the UK, should the border with the Republic of Ireland be dissolved and what in each case should be the rules covering voting for representatives and for constituting a government? And let's suppose that you want to estimate the 'value' to you of the Constitutional Convention. (Maybe, for example, somebody is trying to commit you in advance to accept the outcome of the Constitutional Convention whatever it may be.) How do you set about assigning a 'value' to it?

I'll take it that you have yourself a position on the issue which is identified with that of some bloc of delegates. I'll also assume that you are able to say how much you would like or dislike it if the outcome of the Convention were to correspond to that proposed by each bloc of delegates to the Convention. The 'decisiveness' criterion says that the thing to ask is how likely the bloc with whose position you identify is to be pivotal (Shapley-Shubik) or to be critical to the formation of a coalition (Banzhaf) or to be able to 'uniquely determine' the outcome (Rogowski).

There are two objections to this, of which the first is really a special application of the second.

(1) The 'value' you attach to the Convention will depend crucially on the way the delegates are divided into blocs, even if this doesn't make a significant difference to the outcome expected. Let's suppose, for example, that a majority of the delegates form a bloc committed to some hard-line Protestant position, and that this is the bloc whose position is closest to your own. On any of the three indexes

measuring 'decisiveness', this majority bloc will score unity and the rest zero, if the Convention decides by majority vote.

Now suppose that a splinter bloc splits off from the hard-line Protestant bloc, dedicated to some slight variant in policy, and that you are closer to this bloc's position. If the splinter bloc still leaves the main bloc with a majority of delegates, its score on all three indices will be zero since the main bloc will be pivotal, decisive and uniquely determining. So the 'value' of the Convention to you will drop from one to zero. This seems unreasonable.

(2) Suppose that your favourite faction does have a low or zero probability of being pivotal (etc.), does that matter? It would seem that before you can say, you want to know what the other blocs stand for and what their relative numbers are, and once you do know you won't attach importance to it anyhow. Thus, if your favourite bloc has no chance of changing the outcomes (is a 'dummy') because there is a majority bloc which is going to vote for something close to your preferred position anyway, this is hardly anything to worry about. You'd surely prefer this to a situation in which your favourite bloc would in some combinations be pivotal, necessary or determining but where most combinations yielding a majority of delegate votes would be far from your own preferred position.

I conclude that if you were trying to estimate the 'value' of the Convention, you'd be foolish to bother with the question whether your bloc would be decisive and would ask instead about the outcome to be expected. You would ask first whether any outcome is certain: if so, what's the value of it? If several alternative outcomes seem possible, you'd try to attach a probability to each and then estimate the overall expected value of the Convention by multiplying each probability by its respective value.

A further illustration of the difficulties you get into by following the 'decisiveness' approach is conveniently provided by Rogowski's treatment of 'factions' in his book. In my example of the Northern Ireland Constitutional Convention I've already referred to 'blocs'. I assumed that these were groups of delegates who would always vote the same way between any given pair of proposals. But I didn't go into the question what might lie behind this coincidence of voting.

There are two extreme accounts of voting together by members of a bloc that may be offered:

(1) Blocs are natural (Madison in 10th Federalist)

(2) Blocs are artificial (Rousseau in Social Contract)

(1) Coincidence of voting needs no special explanation but simply follows from the fact that some people are similarly placed (socioeconomically, geographically, etc.) so they will naturally vote together. Voting together is not a result of any sort of discipline but simply arises from each person voting his preferences sincerely.

(2) Opposite extreme: blocs are entirely a product of artifice. They arise from an agreement among a group of people who are no more likely than not to vote the same way if they vote sincerely. The agreement is that they will caucus together before each vote in the main body and they will all agree to cast their votes unanimously as a bloc on whichever side of the question gets a majority in the caucus.

Obviously there is a third view which would be that factions partake of both features together: a group whose members already agree in general may find it expedient to get together and agree to put themselves under discipline so as to increase their collective clout. This is of course the essence of Edmund Burke's definition of a party as a body of men united on some view of the public interest. I'll come back to this a little later.

Rogowski analyses factions purely in the second way -- as the results of agreement among people who are no more likely than not to be initially on the same side of an issue to cast a unanimous vote whichever way a majority of them incline.

Let's follow this up.

Rogowski says that a faction of 50,000 has a very high probability of being decisive in a group of a million and that therefore there's a strong incentive to form one. But from the point of view of an individual, so what? If he's interested in his chance of being on the winning side, then on Rogowski's assumption (that everyone in the society has an <u>a priori</u> probability of voting yes or no on any given issue) his chance of winning is as near as damn it .5 under both circumstances. Admittedly the people outside the faction of 50,000 have less chance of altering the result, but for a given individual within the 50,000 the probability of the result being 'yes' when he's 'yes' is almost the same probability of its being 'yes' when he's 'no'. (The probability for an individual of being decisive is pretty nearly zero both ways -- admittedly much bigger in the group of 50,000 than a million but is this important?)

It's important, I think, to keep our eye firmly on the question of probability of getting the outcome that you want. It's very easy otherwise to get seduced into talking nonsense. Thus, in the literature of 'a priori power' it seems to be assumed that 'forming a coalition' with some other player simply means that you agree to vote together. Brams, p. 173: 'If the two members decide to form a coalition and vote as a bloc...'. But how come? The whole rationale of the computation is that issues come up and the players <u>have</u> positions on them -- not that they <u>choose</u> positions on them. So two players can't simply <u>decide</u> to vote the same way except in the sense that they can decide to vote not in accordance with their preferences, depending on some decision-rule among themselves.

This point reflects what is wrong (or ambivalent) about the notion of 'winning' in Riker. To say you want the side you vote for to win is true if your vote expresses your preference; but it would be silly to vote for the side that you expect to win so as to be able to vote on the winning side. The <u>basic</u> meaning of winning is surely getting the outcome you want -- it's naturally extended to getting the outcome you voted for because it's assumed you vote for what you want. A good general is one who tries to ensure that the side he is on wins -- but does that include changing sides?

What Brams was talking about was not the advantage of forming a faction in the sense of making an agreement to vote together even where your preference for the outcome lies the other way. Rather, what he was saying is that it's lucky if some other people happen to think exactly the same way as you, so that you can in effect always count on some extra votes on your side. This is obviously true but hardly what would naturally be understood by the term 'forming a coalition'.

In fact, it's difficult to see much scope for advantage in a 2-man faction where it's an artificial faction, i.e. where the 2 actors agree to caucus together beforehand.

Take a set of three actors (they could be individuals or they could themselves be the representatives of homogeneous blocs). And suppose each is as likely as not to agree with any of the others.

Natural faction case.

Suppose A and B always happen to be on the same side of every issue. Then of course they always win; if C is as likely as not to agree with them the expectation of each getting the outcome it wants is:

A	1
B	1
C	.5

Artificial faction case.

The obvious prior question here is: what would the rule be for A and B to adopt in their caucus? If both agree anyway on the line to take, there is no problem, but by the same token there is no point in having a caucus for such a case. If on the other hand they disagree, how are they to decide what their joint position shall be?

Rogowski suggests (without explaining why) the rule that if either is No they both vote No. But is this particularly advantageous? We must establish as a baseline what the expectations of A and B would be in the absence of an artificial faction. If we assume that each will agree with the other -- and with C -- half the time, A and B can each expect to get the outcome they want three quarters of the time. (So, of course, can C.) Suppose A is Yes on an issue. B will be Yes half the time; and when B is No, C will be Yes half the time. The case where A is No is exactly similar, so the overall expectation of getting the outcome he wants is 3/4. This is the No Faction Case result.

Now consider A's prospects if he forms an artificial faction with B according to the rule proposed by Rogowski. When he is Yes on an issue, B will be Yes half the time (note that this is an artificial faction) so the outcome will be Yes half the time. (Since A and B always vote together, C can never make a difference to the outcome.) When A is No, the rule prescribed for the caucus says that B will vote No too, so the outcome will always be No. Thus, when A is Yes, he gets a Yes outcome half the time; when A is No, he gets a No outcome all the time. If issues on which he will be Yes and No are equally likely to come up, his overall expectation is $1/2(1/2 + 1) = 3/4$. Thus it is the same (though derived in a different way) as when A and B vote 'sincerely'. There is no gain from forming an artificial faction, unless one supposes that being able to exert a veto is more important than being able to get positive outcomes. (But then a decision rule requiring unanimity among the three would be almost as good: it provides a veto for each and a 1/4 chance of getting a Yes outcome when a given actor is Yes.)

However, if A and B don't gain in their expectation of getting the outcomes they want by forming an artificial faction, C does lose. This is because, in the absence of an artificial faction between A and B, C could in effect decide the outcome where

A and B were on opposite sides. But if A and B always vote No when they are on opposite sides, C's opportunity to break the tie disappears. His probability of getting the outcome he wants falls from 3/4 to 1/2: since he has no influence on the outcome, the probability that the outcome will be the one he prefers is simply the probability that the result of the caucus between A and B will coincide with his preference.

We can set all this out schematically, as follows:

Majority decision-making. No natural factions. A, B, and C vote 'sincerely'.

			Outcome Yes
A Yes	B Yes 1/2		1/2
	B No	C Yes 1/2 x 1/2	1/4
			3/4

Same for A No; cases of B and C like that of A. Expectation (irrespective of relative likelihood of Yes and No) = 3/4.

Decision-making by unanimity. No natural factions. A, B and C vote 'sincerely'.

			Outcome Yes
A Yes	B Yes	C Yes 1/2 x 1/2	1/4
			Outcome No
A No			1

Cases of B and C like that of A. Expectation (if Yes and No equally likely) 1/2(1/4 + 1) = 5/8.

Decision-making by majority. No natural factions. A and B form an artificial faction, with the rule that if A and B are both Yes they both vote Yes, and in all other cases they both vote No.

			Outcome Yes
A Yes	B Yes 1/2		1/2
(A and B vote Yes)			
A No			1
(A and B vote No)			

Case of B same as that of A. If Yes and No equally likely, expectation is 1/2(1/2 +1) = $\frac{3}{4}$

			Outcome Yes
C Yes	A Yes	B Yes 1/2 x 1/2	1/4
	(N.B. A and B must both be Yes)		
			Outcome No
C No	A No		1/2
	A Yes	B No 1/2 x 1/2	1/4
	(N.B. Either A or B must be No)		3/4

If Yes and No equally likely, expectation is 1/2(1/4 + 3/4) = 1/2

Now let us consider the intermediate case (the one I called the Burkean case) where a limited natural affinity between the preferences of A and B is topped up by agreement to form an artificial faction and always vote together. Let us say, therefore, that with 'sincere' voting, A and B would vote together three-quarters of the time and each would be on the same side as C one quarter of the time. There is thus a partial agreement already between A and B and a partial disagreement with C.

In order to establish the effect of an artificial faction between A and B we must first establish the expectations of A, B and C with 'sincere' voting. Clearly, the change in probabilities of agreeing and disagreeing from the case in which each has a one-half chance of agreeing with the others will alter the results in a way favourable to A and B and unfavourable to C.

Partial natural faction between A and B. No artificial faction. Majority voting.

			Outcome Yes
A Yes	B Yes		3/4
	B No	C Yes 1/4 x 1/4	1/16
			13/16

Same if A is No. Expectation is 13/16. Case of B same as that of A.

			Outcome Yes
C Yes	A Yes		1/4
	A No	B Yes 3/4 x 1/4	3/16
			7/16

Same if C is No. Expectation is 7/16.

Now suppose that A and B form an artificial faction according to the rule already canvassed: that when both are Yes they will vote Yes but if either or both are No both will vote No.

		Outcome Yes
A Yes	B Yes	3/4
		Outcome No
A No		1

Expectation is 1/2(3/4 + 1) = 7/8.

The case of B is the same as that of A.

			Outcome Yes
C Yes	A Yes	B Yes 1/4 x 1/4	1/16
			Outcome No
C No	A No		1/4
	A Yes	B No 3/4 x 1/4	3/16
			7/16

C's expectation is 1/2(1/16 + 7/16) = 1/4.

In summary form:

	Expectation		
	A	B	C
No artificial faction	13/16	13/16	7/16
Artificial faction between A and B	7/8	7/8	1/4

Here, as we can see, there is a definite gain to A and B from forming an artificial faction, as well as a loss to C. Thus, so far it would appear that there is more point in forming an artificial faction with those whom one already tends to agree with. The converse of this, it may be noted, is that it may be worse to form an artificial faction with those who disagree with you on balance than to form no faction at all. Suppose A and C form an artificial faction in the kind of case we have just been considering, applying the same rules for their caucus. A and C agree one quarter of the time; so when A is Yes, C is Yes a quarter of the time.

		Outcome Yes
A Yes	C Yes	1/4
		Outcome No
A No		1

A's expectation is 1/2(1/4 + 1) = 5/8.

C's case is the same as A's.

The result should be compared with that where there is no artificial faction. C gains from the artificial faction: instead of finding himself most of the time in the minority because of the tendency of A and B to agree, he has eliminated B from any influence on outcomes and has parity with A. He moves up from an expectation of 7/16 to one of 5/8. A, on the other hand, loses: he has thrown away the advantage of tending to agree with B. He moves down from an expectation of 13/16 in the absence of an artificial faction to 5/8.

Finally, how does B fare?

			Outcome Yes
B Yes	A Yes	C Yes 3/4 x 1/4	3/16
			Outcome No
B No	A No		3/4
	A Yes	C No 1/4 x 1/4	1/16
			13/16

B's expectation is 1/2(3/16 + 13/16) = 1/2.

Thus, B also comes down -- even further than A.

The case of a decision-making body with three members is the simplest in which we can display the phenomenon of an artifical faction at all. But it is an atypical case because there is no room for majority decision-making in a faction with only two members. To get that we have to move up to a faction of three members, and if we want to avoid the complication of ties that means a decision-making group of five members.

The computations become a good deal more complex when we move from three to five, but the same kinds of result hold. However, unlike the case of a two-man artificial faction where the members have a one-half probability of agreeing with one another, there is a small advantage to the members from forming a three-man artificial faction.

We should begin again by looking at the case where there is no artificial faction and each of the five is equally likely to agree or disagree with any other:

<u>When A is Yes:</u>

B	C	D	E	Outcome Yes
Yes 1/2	Yes 1/2×1/2			1/4
	No 1/2×1/2	Yes 1/2×1/2×1/2		1/8
		No 1/2×1/2×1/2	Yes 1/2×1/2×1/2×1/2	1/16
No 1/2	Yes 1/2×1/2	Yes 1/2×1/2×1/2		1/8
		No 1/2×1/2×1/2	Yes 1/2×1/2×1/2×1/2	1/16
	No 1/2×1/2	Yes 1/2×1/2×1/2	Yes 1/2×1/2×1/2×1/2	1/16
				11/16

Same for No. Expectation is 11/16 (.69). B, C, D and E same case.

Now suppose that A, B and C agree to form an artificial faction, voting as a bloc whichever way the majority of them prefer. The situation is then in effect one in which A, B and C are the only voters. (The caucus vote always turns into the majority vote in the five-man decision-making body.) We already know that in a 3-man group each member gets the outcome he wants three-quarters of the time. So the expectation of A, B and C rises as a result of their forming an artificial faction from 11/16 to 3/4 -- a gain of one-sixteenth.

D and E, however, lose more than A, B and C gain. Their expectation is the same as it would be if they were facing a single majority player who is as likely to agree or disagree with them. They drop from an expectation of 11/16 to one of 1/2.

It's important to notice that although the group of 3 out of 5 has complete power collectively, the gain to each member is still quite small. Now suppose that A, B and C expect to agree 3/4 of the time with each other and 1/4 with D and E. First, what ahppens without a 'faction' (i.e. each simply votes 'sincerely')?

A Yes.

B	C	D	E	Outcome Yes
Yes 3/4	Yes 3/4×3/4			9/16
	No 3/4×1/4	Yes 3/4×1/4×1/4		3/64
		No 3/4×1/4×3/4	Yes 3/4×1/4×3/4×1/4	9/256
No 1/4	Yes 1/4×3/4	Yes 1/4×3/4×1/4		3/64
		No 1/4×3/4×3/4	Yes 1/4×3/4×3/4×1/4	9/256
	No 1/4×1/4	Yes 1/4×1/4×1/4	Yes 1/4×1/4×1/4×1/4	1/256
				187/256 = 0.73

The case where A is No is the same, so A's expectation is .73. B and C are in the same position.

What about the view from D or E?

D Yes

A	B	C	E	Outcome Yes
Yes 1/4	Yes 1/4×1/4			1/16
	No 1/4×3/4	Yes 1/4×3/4×1/4		3/64
		No 1/4×3/4×3/4	Yes 1/4×3/4×3/4×3/4	27/256
No 3/4	Yes 3/4×1/4	Yes 3/4×1/4×1/4		3/64
		No 3/4×1/4×3/4	Yes 3/4×1/4×3/4×3/4	27/256
	No 3/4×3/4	Yes 3/4×3/4×1/4	Yes 3/4×3/4×1/4×3/4	27/256
				113/256 = 0.44

Now suppose A, B and C form an alliance to vote as any two of them prefer. The outcome now depends only on the preferences of A, B and C.

A Yes.			Outcome Yes
	B	C	
	Yes 3/4		3/4
	No 1/4	Yes $1/4 \times 3/4$	3/16
			15/16 = 0.94

Thus, A, B and C gain greatly by forming a coalition here and voting in accordance with majority rule.

How do D and E fare under this arrangement?

D Yes.				Outcome Yes
	A	B	C	
	Yes 1/4	Yes $1/4 \times 1/4$		1/16
		No $1/4 \times 3/4$	Yes $1/4 \times 3/4 \times 1/4$	3/64
	No 3/4	Yes $3/4 \times 1/4$	Yes $3/4 \times 1/4 \times 1/4$	3/64
				10/64

D or E have a probability of only 0.16 that the outcome will coincide with what they want, as against 0.44 in the situation where there are no factions.

As before, note that it's important to get in a group of like-minded people. If you get into a group with people whom you expect will <u>disagree</u> with you, you'd be better off staying outside.

Thus, as we saw, in the absence of factions, A has a .73 chance of getting his preferred outcome, given that B and C have a .75 probability of agreeing with him and D and E a .25 probability of agreeing with him.

Now suppose that A forms an artificial faction with D and E. How likely is the 3-man dicisive group likely to cast its 3 votes the way he wants?

A Yes		Outcome Yes
D	E	
Yes 1/4		1/4
No 3/4	Yes 3/4×1/4	3/16
		7/16

Thus, given that A is Yes, the outcome will be Yes only .48 of the time! But A could have had a probability of 0.73 of getting what he wanted in the situation where there were no factions at all. However, note that for D and E the result is an improvement: they now have an expectation of .81 which compares exceptionally favourably with their .44 expectation in the absence of any factions.

If D is Yes:		Outcome Yes
A	E	
Yes 1/4		1/4
No 3/4	Yes 3/4	9/16
		13/16 = 0.81

What about B and C?

If B is Yes:			Outcome Yes
A	D	E	
Yes 3/4	Yes 3/4×1/4		3/16
	No 3/4×3/4	Yes 3/4×3/4×1/4	9/64
No 1/4	Yes 1/4×1/4	Yes 1/4×1/4×1/4	1/64
			11/32 = 0.34

B and C thus do particularly badly, falling from an expectation of .73 with no artificial factions to an expectation of .34 when A forms an artificial faction with D and E.

We can summarize the results for the case of a five-man decision-making body reaching its decisions by majority vote in the following form:

Each agrees with others half the time:

	A	B	C	D	E
No artificial faction	.69	.69	.69	.69	.69
Artificial faction between A, B and C	.75	.75	.75	.5	.5

A, B and C agree 3/4 of the time with each other, 1/4 of the time with D and E:

	A	B	C	D	E
No artificial faction	.73	.73	.73	.44	.44
Artificial faction between A, B and C	.94	.94	.94	.16	.16
Artificial faction between A, D and E	.48	.34	.34	.81	.81

The upshot of this analysis is then, first, that it's of course best to have people who naturally form a faction with you (i.e. vote the same way without any prearrangement) because you don't have to compromise with them. In the case of a 5-man committee, if A, B and C form a natural faction (i.e. always agree spontaneously) they of course always get what each wants.

Second, there's some advantage in forming an artificial faction even with people you don't expect to agree with more often than you expect to agree with those outside. But the advantage is greater if you agree with them naturally to some extent. Thus, the Burkean conception of a party stands up as well founded.

So much for abstract analysis. Let us now turn to some political questions involving 'rational legitimacy' and put the apparatus to work. I shall take up one general question involving the degree of conflict produced by different cleavage structures and then deal with a concrete example: the US Constitutional Convention

(1) Joining a Bloc

Simmel spent a lot of time distinguishing between the inherent characteristics of dyads and triads. Recently this line of speculation has been revived independently in relation to rational legitimacy. Rose, in *Governing Without Consensus*, has suggested that it makes a great deal of difference to the stability of a political system containing groups with divergent goals or interests (typically based on ethnicity, language, religion, race etc.) whether there are two or whether there are at least three. More precisely, his idea is that it's important that one group shouldn't form a majority.

It seems to me that our analysis suggests that the problem for stability arises out of some group's being on the losing side too often. (Whatever 'too often' may be.) And we can't say that it's necessarily worse to be in a minority facing a majority than as one of three groups none of which has a majority.

It's true that the *worst* outcome you can get as a minority in a 2-bloc situation is worse than anything you can get in a situation where there are 3 blocs none of whom has a majority. But this is mathematically trivial. If one other bloc, it can be on the other side of every issue, but if there are 3, it's not possible that *both* of them can be on the other side of *every* issue -- they'd have to be identical. If one bloc which is in the majority disagrees with you half the time, you get the outcome you want half the time. If two other blocs *each* disagree with you half the time (and with each other half the time) you get what you want three quarters of the time (assuming that the set-up is one in which any two blocs constitute a majority and no single bloc does). If the other blocs *each* disagree with you 3/4 of the time, you get what you want only 7/16 of the time.

Suppose you are A. When A is Yes:

B	C	Outcome Yes
Yes 1/4		1/4
No 3/4	Yes 3/4×1/4	3/16
		7/16

This is, of course, slightly <u>worse</u> than having a majority bloc disagreeing with you half the time.

We could also look at Eric Nordlinger's suggestion that in a 2-bloc case, mutual veto is a way of creating legitimacy where majority decision-making is liable to destroy it. Whether mutual veto improves the expectations of the minority group depends on the extent to which the minority will be on the opposite side of issues to the majority group.

Suppose A (the minority group) anticipates agreement by B 1/2 the time.

	A	B		
			<u>Outcome Yes</u>	
Majority Voting	Yes	Yes	1/2	Expectation:
			<u>Outcome No</u>	1/2(1/2 + 1/2) = 1/2
	No	No	1/2	
			<u>Outcome Yes</u>	
Veto	Yes	Yes	1/2	Expectation:
			<u>Outcome No</u>	1/2(1/2 + 1) = 3/4
	No		1	

Now suppose A expects B to agree only 1/4 of the time.

	A	B		
			<u>Outcome Yes</u>	
Majority Voting	Yes	Yes	1/4	Expectation:
			<u>Outcome No</u>	1/2(1/4 + 1/4) = 1/4
	No	No	1/4	
			<u>Outcome Yes</u>	
Veto	Yes	Yes	1/4	Expectation:
			<u>Outcome No</u>	1/2(1/4 + 1) = 5/8
	No		1	

Finally, suppose A expects B <u>always</u> to disagree.

	A	B		
			<u>Outcome Yes</u>	
Majority Voting	Yes	Yes	0	Expectation:
			<u>Outcome No</u>	1/2(0 + 0) = 0
	No	No	0	
			<u>Outcome Yes</u>	
Veto	Yes	Yes	0	Expectation:
			<u>Outcome No</u>	1/2(0 + 1) = 1/2
	No		1	

Thus, the more the minority group disagrees with the majority (the more deeply the society is divided) the greater the advantage to it of replacing majority decision-making by mutual veto (decision-making by unanimity).

We can show this in summary form:

Probability of agreement between majority and minority group	Minority group's probability of getting desired outcome	
	Under majority decision-making	Under mutual veto
3/4	3/4	7/8
1/2	1/2	3/4
1/4	1/4	5/8
0	0	1/2

Thus the <u>gain</u> is 1/8 in the first case but rises to 1/2 in the last case.

A further point to observe is that if a minority group does not know how opposed the majority group will be, a minimax strategy will entail going for a mutual veto. More strongly (since if the group knows it will be in a minority, veto dominates majority decision-making), if a group does not know whether it will be in a majority of not, a minimax strategy entails going for a mutual veto.

(2) <u>The U.S. Constitutional Convention of 1789.</u>

The one seriously divisive issue -- the only one on which there was any prospect of a breakdown in the deliberations -- was whether in the Senate the States should each have the same representation or whether representation should be proportional to population, as in the House of Representatives. We know what the initial proportions were to be in the House of Representatives, since they formed part of the Constitution, so the delegates were aware exactly what was at stake. The numbers provided for in the House were as follows (p. 617 of Madison's <u>Notes of Debates</u>):

Va	10	Va, Mass, Penn	26
Mass, Penn	8		
NY, Md	6	Others	39
Conn, NC, SC	5		
NJ	4		
NH, Ga	3		
RI, Del	1		
		Total	65
		(Majority	33)

It seems clear that the smaller States had apprehensions that, if both Houses of Congress were allocated according to population, the three largest would be able virtually to control federal law-making.

Madison did his best to allay these fears in one of the few extended speeches of his own that are reported. In part, this went as follows:

> That it is not necessary to secure the small States agst the large ones he conceived to be equally obvious: Was a combination of the largest ones dreaded? this must arise either from some interest common to V^{a} Mas^{ts} & P^{a} distinguishing them from the other States or from the mere circumstance of similarity of size. Did any such common interest exist? In point of situation they could not have been more effectually separated from each other by the most jealous citizen of the most jealous State. In

> point of manners, Religion, and the other circumstances which sometimes beget affection between different communities, they were not more assimilated than the other states.——In point of the staple productions they were as dissimilar as any three other States in the Union. The Staple of Mas^{ts} was fish, of P^{a} flower, of V^{a} Tob^{o}. Was a combination to be apprehended from the mere circumstance of equality of size? Experience suggested no such danger. The journals of Cong^{s} did not present any peculiar association of these States in the votes recorded. It had never been seen that different Counties in the same State, conformable in extent, but disagreeing in other circumstances, betrayed a propensity to such combinations. Experience rather taught a contrary lesson. Among individuals of superior eminence & weight in Society, rivalships were much more frequent than coalitions. Among independent nations, pre-eminent over their neighbours, the same remark was verified. Carthage & Rome tore one another to pieces instead of uniting their forces to devour the weaker nations of the Earth. The Houses of Austria & France were hostile as long as they remained the greatest powers of Europe. England & France have succeeded to the pre-eminence & to the enmity. To this principle we owe perhaps our liberty. A coalition between those powers would have been fatal to us. Among the principal members of antient & Modern confederacies, we find the same effect from the same cause. The contintions, not the Coalitions of Sparta, Athens & Thebes, proved fatal to the smaller members of the Amphyctionic Confederacy. The contentions, not the combinations of Prussia & Austria, have distracted & oppressed the Germanic empire. [pp. 205-206]

This is an interesting case in that it is one in which it would appear that the participants were quite explicitly posing the question whether or not to join a political association under such-and-such rules in terms of 'rational legitimacy' -- that is to say, on my interpretation of the concept, in terms of their likelihood of finishing up with outcomes corresponding to those that they wanted. Let us apply our machinery to it.

First of all, how true would it have been that the 'big three' (Virginia, Massachusetts and Pennsylvania) would have been virtually certain to carry any policy which all their representatives supported? Obviously, before we can make any estimate of this we must postulate some probability of each other State's representatives voting the same way as the 'big three'. Let us make the 'neutral' assumption that when the 'big three' vote together any other State's representatives have a .5 probability of voting the same way. Further, let us assume that between the representatives of any two States outside the 'big three' there is a .5 probability of being on the same side on an issue where the 'big three' vote together. This last proviso is important. Suppose that each of the other States voted on the same side as the 'big three' half the time, but that the other States always voted the same way as one another. The 'other' States would then constitute a majority bloc which would always get what it wanted. The 'big three' would get what they wanted half the time, but only in virtue of wanting the same thing as the 'others' half the time. The 'big three' would be powerless to affect outcomes.

Take, then, the case where the 'big three' are united and the other States fragmented -- as likely to agree with one another and with the 'big three' as not. What is the probability that the outcome will be Yes when the 'big three' vote Yes and No when the 'big three' vote No? The answer is 0.973, in other words they would be defeated only three times in a hundred votes. What about the expectations of the 'other States'? A State which had no vote or, because of the peculiarities of the distribution of votes, could never change the outcome from what it would be on the basis of the votes of the remaining States, would have a .5 chance of getting an outcome it would like. This follows from the .5 probability any State has of agreeing with any other. To the extent that a State has the possibility of altering an outcome by the direction of its vote, its probability of getting the outcomes it favours rises above .5. However, the range of increases is not very great. For New York or Maryland, the largest of the 'other States', the probability of getting a Yes outcome when the State's representatives vote Yes or a No outcome when they vote No is 0.525. For Rhode Island or Delaware, the smallest of the 'other States', the comparable figure is 0.506.

No doubt if we concentrate on the increase we can say that the difference is considerable. Rhode Island can raise its chance of getting an outcome it wants by .006 over the .5 it would have if it had no vote at all, whereas New York can raise it by .025 -- over four times as much. (It may be recalled that New York has six votes to Rhode Island's one.) But the point to emphasize is that even .025 is a tiny amount. It would be swamped by a relatively small downward adjustment in the estimation made of the probability of agreeing with the 'big three'. For example, it would obviously be preferable from the standpoint of 'rational legitimacy' to be Rhode Island with its small addition to the basic probability of coinciding with the 'big three' but with a basic probability of .5 of agreeing with them than to be New York with its larger addition to the basic probability of coinciding with the 'big three' if that basic probability were, say, .45. Rhode Island's probability of getting a desired outcome would be .506 while New York's would be .479.

This consideration should head off any recrudescence of the idea that 'power' (in the sense of ability to alter the outcome) should be the basis of 'rational legitimacy'. The significance of variations in power is liable to be smaller than even minor variations in probability of agreeing with actors. Thus, if New York expected <u>always</u> to be on the opposite side to the 'big three', while agreeing half the time with the rest (and the rest agreeing half the time with each other) its expectation of getting the outcomes it wanted would be only .0527; the comparable figure for Rhode Island would be .0332.

Even more, the probability of being able to change the outcome by one's vote is itself a function of the estimates of agreement with others. And, to repeat a point made in general terms earlier, it may be that power -- in the sense of ability to change the outcome by one's vote -- increases as the probability of getting an outcome one wants decreases. This can conveniently be illustrated by observing that if all the States except the 'big three' always voted together each member would always get the outcome it wanted, however the 'big three' were to vote -- even if the 'big three' were always on the other side. But since the majority over the 'big three' would be 39-26 -- a margin of thirteen votes -- no single member of the 'other' States could make a difference to the outcome by casting its vote differently. Even if New York or Maryland were to change sides, the result would be a 35-30 vote in favour of the 'others'. Thus no single State would have any 'power', because the outcome would be the same however it voted. This is not a trivial point: suppose New York did decide that its interests lay with the 'big three', it would always go down to defeat with them.

It may be helpful to understanding the logic of the analysis to see in some little detail the way in which the probability figures given above are arrived at. We start with the figure given first: the expectation of the 'big three' that if they vote together the outcome will be the one they want. This is derived by looking at the contingencies under which if the 'big three' all vote Yes the outcome can be No. Thus, for example, if New York with its six votes joins the 'big three' with their 26 votes, making 32 Yes votes out of 65, the only way in which the outcome can still be No is if all the remaining States vote No. This is the contingency represented in the first row. Since it is a contingency that requires ten States to vote a certain way and each has a .5 probability of voting that way, the probability of the contingency's arising when the 'big three' vote Yes is $1/2^{10} = 1/1024$. The third row, on the other hand, shows that if the first seven 'other' States vote No the outcome will be No whatever the last three States do because the first seven 'other' States dispose of 34 votes between them, and this is a majority. There are therefore eight contingencies covered by this case, since the question mark (meaning 'Yes or No') can be filled in to give eight combinations of the votes of the three remaining States. Counting up all the contingencies gives us 28 ways in which a No outcome can occur when the 'big three' vote Yes, out of 1024 possible ways in which the ten 'other' States could vote. This is 0.0273.

	$\frac{1}{2}$ NY (6)	$\frac{1}{4}$ Md (6)	$\frac{1}{8}$ Conn (5)	$\frac{1}{16}$ NC (5)	$\frac{1}{32}$ SC (5)	$\frac{1}{64}$ NJ (4)	$\frac{1}{128}$ NH (3)	$\frac{1}{256}$ Ga (3)	$\frac{1}{512}$ RI (1)	$\frac{1}{1024}$ Del (1)	Outcome No. (in 1024^{ths})	Row No.
Va,	Yes	No	No	No	No	No	No	No	No	No	1	1
Mass,	No	Yes	No	No	No	No	No	No	No	No	1	2
Penn			No	No	No	No	No	?	?	?	8	3
Yes			No	No	No	No	Yes	No	?	?	4	4
(26	NY		No	No	No	Yes	No	No	?	?	4	5
votes)	and		No	No	Yes	No	No	No	No	?	2	6
	Md		No	Yes	No	No	No	No	No	?	2	7
	both		Yes	No	No	No	No	No	No	?	2	8
	No		No	No	No	No	Yes	Yes	No	No	1	9
			No	No	Yes	No	No	No	Yes	No	1	10
			No	Yes	No	No	No	No	Yes	No	1	11
			Yes	No	No	No	No	No	Yes	No	1	12
											28	

28/1024 = 0.0273

This is to look at things from the perspective of the 'big three'. If they ask 'When we vote for something what is the probability that the outcome will correspond with what we want?' the answer is 1 — 0.027 = 0.973.

But from the perspective of one of the 'other' States, let us call it State X, the question to be asked is 'Given that the 'big three' vote together, what is the probability that the outcome will be the one that State X wants?' Clearly, there are two cases, which we are assuming at the moment are equiprobable: (1) on a given issue State X is on the same side as the 'big three' and (2) on a given issue State X is on the opposite side. So the question to be asked by State X can be broken down into two sub-questions. (1) What is the probability that the outcome will be the one X prefers when the 'big three' vote on the same side? And (2) What is the probability that the outcome will be the one X prefers when the 'big three' vote on the opposite side.

Let us take up these two questions from the point of view of New York.

(1) Row one of the table tells us that if New York joins the 'big three' in voting Yes the only way in which the outcome can be No is when all the other nine States vote No. (The case where New York joins the 'big three' in voting No is exactly symmetrical so we do not need to consider it separately.) The probability of nine 'other' States all voting the same way (and one particular way) is $1/2^9$ or 1/512 (approximately 0.002). So the probability that New York will not get its desired outcome when it is on the same side as the 'big three' is 1 — 0.002 = 0.998.

(2) Now we have to suppose that New York is No when the 'big three' are Yes (or vice versa, of course, but this way round corresponds to the way the table is set up). Under how many contingencies will the outcome be No? We can see from the table that rows 2-12 cover all the cases where New York is No when the 'big three' are Yes and the outcome is No. (Row 1 is an irrelevant case for the present purpose.) These, as we can see from the table, would comprise 27 contingencies out of 1024 if New York had a .5 probability of voting No. But <u>ex hypothesi</u> the present case is

one where New York is definitely voting No. The number of contingencies is not $1/2^{10}$ but $1/2^{9}$. The relevant probability is therefore 27/512 = 0.0527.

Since we are assuming that New York has an a priori probability of agreeing and disagreeing with the 'big three' half the time when they vote together, we can put together these two values to get New York's expectation of getting the outcomes it wants: this is $(.5 \times 0.998) + (.5 \times 0.0527) = .499 + .0264 = .525$.

We may now compute, at the other extreme in size among the 'other' States, the comparable probabilities for Rhode Island or Delaware. Since Delaware is the last State appearing in the table it is more convenient to carry out the analysis in terms of it. We again start with the case of agreement with the 'big three'.

(1) We know from the table that there are six contingencies in which a No from Delaware is necessary to make the outcome No when the 'big three' vote Yes (rows 1, 2, 9, 10, 11 and 12). When Delaware votes Yes along with the 'big three' these six contingencies are ruled out. Instead of 28 contingencies there are now only 22 which allow for a No vote. The other six rows (which account for these 22 contingencies) are, as indicated by the question marks in the last column, cases where it does not make any difference to the No result which way Delawar votes. The probability that there will be a No outcome when Delaware joins the 'big three' in voting Yes is therefore 22/1024 (0.0215). The probability that Delaware will get the outcome it wants when it is on the same side as the 'big three' is thus $1 - 0.0215 = 0.9785$.

(2) What is the probability that the outcome will be No when Delaware votes No but the 'big three' vote Yes? Again, we can obtain the answer by referring to the table. In the six contingencies where Delaware makes a difference to the outcome (rows 1, 2, 9, 10, 11 and 12) we can say that the probability of Delaware's voting No is not a half (as is assumed in the table) but that it will certainly vote No.

Each of these contingencies therefore occurs 1/512 of the time rather than 1/1024 of the time. This ($1/2^9$) is the probability that the other nine States will vote in the way indicated in these rows. The 28/1024 figure thus rises to 34/1024 (0.0332). The expectation for Delaware is thus $(.5 \times 0.9785) + (.5 \times 0.0332) = 0.489 + 0.0166 = 0.5056$.

It can now be seen that it is a simple matter to substitute different expectations for State X of its likelihood of finding itself on the same side as the 'big three' when the 'big three' vote as a bloc. The contingent probabilities entering into the calculation do not require to be recomputed provided we leave intact the assumption that the <u>other</u> parties than X still continue to have an equal chance of agreeing with one another and with the 'big three' when the 'big three' vote together. Let us illustrate by taking, in addition to a .5 probability of X agreeing with the 'big three' the cases of 0, .25, .75 and 1.

	NEW YORK			DELAWARE			DUMMY STATE		
Probability of agreeing with 'big three'	New York Yes when 'big three' Yes	New York Yes when 'big three' No	Expectation of Yes outcome when New York Yes	Delaware Yes when 'big three' Yes	Delaware Yes when 'big three' No	Expectation of Yes outcome when Delaware Yes	Dummy State Yes when 'big three' Yes	Dummy State Yes when 'big three' No	Expectation of Yes outcome when Dummy State Yes
1	1×.998 =.998	—	.998	1×.979 =.979	—	.979	1×.9727 =.9727	—	.973
.75	.75×.998 =.749	.25×.0527 =.0132	.762	.75×.979 =.734	.25×.0332 =.0083	.742	.75×.9727 =.7295	.25×.0273 =.0068	.736
.5	.5×.998 =.499	.5×.0527 =.0264	.525	.5×.978 =.489	.5×.0332 =.0166	.506	.5×.9727 =.48635	.5×.0273 =.01365	.5
.25	.25×.998 =.2495	.75×.0527 =.0395	.289	.25×.979 =.2448	.75×.0332 =.0249	.270	.25×.9727 =.2432	.75×.0273 =.020475	.264
0	—	1×.0527 =.0527	.0527	—	1×.0332 =.0332	.0332	—	1×.0273 =.-273	.0273

In the table we have considered three States: New York (with six votes), Delaware (with one vote) and a hypothetical Dummy State with no vote. (With bloc voting systems it can sometimes happen that one or more of the voters are 'dummies', in the sense that they can never change the result however the other votes are cast, even though they do have votes.) The Dummy State is, in the conventional sense, powerless: nothing it can do will make any difference to what happens. (We are assuming here that voting is the only way of getting outcomes -- as against, say, bribing or coercing those who do have votes. But the analysis could be extended to allow for other forms of power. We could still imagine a State which was a helpless observer of outcomes with no ability to influence them in any way whatsoever.)

The Dummy State provides a baseline against which we can assess the advantage to a State of having one or more votes. As will be seen, the advantage of having even six votes out of a total of 65 is relatively small. Thus, a Dummy State that expected always to be on the opposite side to the 'big three' would get the outcomes it wanted 2.7% of the time. (This is, of course, simply the probability that we began with: the probability that the 'big three' will fail to get something when they all vote for it.) Delaware, with one vote, would get the outcome it wanted if it were always in opposition to the 'big three' 3.3% of the time, and New York, with six votes, 5.3% of the time. Clearly, the expectation of agreeing with the 'big three' is much more important than the difference between no vote, one vote or six votes. Thus the Dummy State, if it expected always to agree with the 'big three' would get its desired outcome 97.3% of the time, Delaware would get it 97.9% of the time and New York would get it 99.8% of the time.

We can put a figure on New York's advantage derived from its six votes by comparing New York with the Dummy State. For each level of probability of agreeing with the 'big three' we can look at the Dummy State's probability of getting the outcome it wants and subtract it from New York's to give the increment provided to New York by its six votes -- New York's 'power' to change the outcome by its active participation.

Probability of agreement with 'big three'	% of desired outcomes		
	Dummy	New York	Increment
1	97.3	99.8	2.5
.75	73.6	76.2	2.6
.5	50.0	52.5	2.5
.25	26.4	28.9	2.5
0	2.7	5.3	2.6

Thus, ignoring errors due to rounding, we can say that New York's six votes obtain an increment of 2.5% in the number of times it can expect to get the outcome it wants, at any level of probability of agreement with the 'big three'. This is not to be sneezed at but it is clearly much less important than the likelihood of agreeing with the 'big three'.

It may, perhaps, be said that you don't know, when entering some association, how your own preferences or interests will change or how those of the other participants will change, and that therefore one should stick to a calculation of 'power'. There are three answers to this. First, if nobody joins any association unless he has enough 'power' to get the outcomes he wants most of the time <u>whatever</u> the positions of the other participants, there could be no associations. Second, if one is wondering whether to join an association in which he has relatively little 'power', on most combinations of others' preferences, even a rough guess about the likelihood of finding a majority on his side is much more relevant than the amount of 'power' he will have. And, third, the calculation of 'power' itself depends upon the expectations one has of the way others will vote. If a majority not including X is always going to vote as a bloc, X is 'powerless' however many votes he has (and however likely or unlikely he is to agree with that majority bloc). On the other hand, if the other participants could be counted on always to divide exactly evenly, X would be 'all-powerful' provided he had even one vote.

The standard power indexes like those of Shapley-Shubik and Banzhaf are each computed on the basis of one particular assumption about the way in which the votes of other participants will distribute themselves: Shapley-Shubik that all permutations of votes are equally probable, Banzhaf that all combinations of minimum winning coalitions are equally probable. These are special assumptions neither of which has anything much to commend it. Even the roughest guess about the likely actual situation would be superior.

Almost everything that has been said so far presupposes that the 'others' -- except the one State we are considering at a given time -- have a .5 probability of voting on the same side as one another and the 'big three'. We can of course vary this assumption. We have already noted that the 'others' might all vote together. If they always voted the opposite way to the 'big three', the 'big three' would never get the outcomes they wanted. As an intermediate case, we might consider the one where all the 'others' have a .75 probability of agreeing together and each has a .25 probability of agreeing with the 'big three' (who, we continue to assume, vote together). This requires a recalculation -- the results cannot be derived directly from previously stated results. To give an example, New York would under this arrangement have an expectation of getting an outcome it wanted of .605. This is obviously better than the .525 it could expect if all the 'others' were as likely to agree with one another (and the 'big three') as not. Thus, any tendency for the 'others' to vote the same way is beneficial to all of them.

All this analysis, in spite of its intricacy, has done no more than scratch the surface. We have throughout premised our discussion on the assumption that the 'big three' would vote as a bloc and have asked what the implications of that would be, especially for the 'rational legitimacy' of one of the 'other' States. Obviously, we could ask many other questions using the same apparatus. I hope, though, that the analysis carried out suggests the utility of thinking in the way suggested.